

Kaplan Schweser 2017 FRM Part I 10-Week Online Class Week 2



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Getting Up to Speed on the Financial Crisis: A One- Weekend-Reader's Guide

Topic 8



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Financial Crisis Overview

- The main *trigger* of the financial crisis was the prospect of losses on subprime mortgages
- In the first half of 2007, housing prices in the U.S. started to decline, causing several subprime mortgage lenders to file for bankruptcy
- These losses became amplified as they had a ripple effect that spread to the main *vulnerabilities* of the crisis, asset-backed commercial paper (ABCP), and repurchase agreements (repos)

Lehman Brothers Failure

- The Lehman Brothers bankruptcy filing in September 2008 is considered the tipping point in the financial crisis
- It eroded confidence and caused a run on money market mutual funds (MMFs)
- The lack of confidence spread across markets and countries, amplifying losses in the subprime mortgage market

Previous Financial Crises

- The recent financial crisis was not unique compared to previous banking crises
- It followed a similar pattern of increased public and private debt, increased credit supply, and increased housing prices preceding and leading to the crises
- Studies showed that as the recent crisis strengthened, the demand for credit decreased

Panic Periods

- The two main panic periods of the financial crisis were August 2007 and September through October 2008
 1. The *first panic* period in August 2007 occurred when there were runs on ABCP
 2. The start of the *second panic* period was September 2008 when Lehman Brothers filed for bankruptcy

Panic Periods (continued)

- Lehman's failure caused a run on a particular MMF called Reserve Primary, which contained commercial paper issued by Lehman
- The run on Reserve Primary spread to other MMFs, which started a contagion effect that spread to other assets that were falling in price in tandem with rising haircuts

Government Policy Responses

- The International Monetary Fund (IMF) studied 13 developed countries and their responses to the financial crisis
- This study found 153 separate policy actions that were divided into 5 subgroups consisting of: (1) interest rate changes, (2) liquidity support, (3) recapitalization, (4) liability guarantees, and (5) asset purchases

Government Policy Responses (continued)

- To measure the impact of interest rate cuts, the IMF used the economic stress index (ESI) and the financial stress index (FSI)
- Liquidity support was measured using interbank spreads and the FSI
- Recapitalization, liability guarantees, and asset purchases were measured using the FSI and an index of credit default swaps (CDSs) on banks

Government Policy Responses (continued)

- The evidence of the study suggested that the *most effective* measures taken were the **liquidity support** stabilizing the interbank markets before the Lehman failure and **recapitalization** (capital injections), which was considered the most effective tool after the Lehman failure

Risk Management Failures: What Are They and When Do They Happen?

Topic 9



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Book 1, LO 9.1

The Role of Risk Management

- Risk management involves assessing, communicating, monitoring, and managing risks
- A large loss does not necessarily mean that risk management has failed

The Role of Risk Management (continued)

- Risk management can fail if the firm does not:
 - Measure risks correctly
 - Recognize some risks
 - Communicate risks to top management
 - Monitor and manage risks
 - Use appropriate risk metrics

Risk Mismeasurement

- Mismeasurement can occur when management does not understand the distribution of returns of a single position or the relationships of the distributions among positions and how the distributions and correlations can change over time
- Mismeasurement can also occur when managers must use subjective probabilities for rare and extreme events

Failure to Account for Risk

- Failing to take known and unknown risks into account can take three forms:
 1. Ignoring a risk that is known
 2. Failing to properly incorporate a risk
 3. Not finding all risks

Ignoring Risk

- A firm ignores known risks by failing to realize how various position risks can lead to a potential disaster
- This was the case when Long-Term Capital Management (LTCM) failed to recognize that high-yielding Russian debt had not only default risk, but also currency risk, sovereign risk, and counterparty risk

Failing to Incorporate Risk

- Not collecting and entering data into the appropriate risk models is another potential source of disaster
- In this case, the firm may make an attempt to recognize the risk
- Not obtaining proper data to measure the risk adequately will have similar consequences to ignoring risks

Failure to Account for Risk

- One of the severe consequences of either ignoring or not adequately using data is that the firm might expand its operations in areas where risks are not being properly accounted for
- Another risk that is often ignored is increasing correlations during a time of crisis
 - Not recognizing the possibility of increasing correlations could potentially lead to large losses

Not Finding All Risks

- Some risks may go completely undetected by risk managers
 - Clearly, the same unfavorable outcomes discussed previously would result
 - In some cases, however, unknown risks may not be too severe of a problem
- As long as management realizes that not all risks will be known and makes appropriate capital allocations to account for this, then unknown risks may not be a severe problem

The Role of Risk Metrics

Risk metrics aid the management process by providing managers a target to achieve

- Monitoring these risk metrics allows managers to appropriately manage risk
- However, risk metrics may be too narrow in scope, which can make it more difficult to achieve the overall objective of managing risk in an effort to create value
 - For example, VaR usually assumes independent losses across periods of time

The Role of Risk Metrics (continued)

- Risk metrics generally fail to capture the effect of a firm's actions on the overall market and behavior patterns such as **predatory trading**
 - Predatory trading occurs when other firms in a market see that a large player in the market is in trouble
 - The other firms attempt to push the price down further in order to hurt the large player

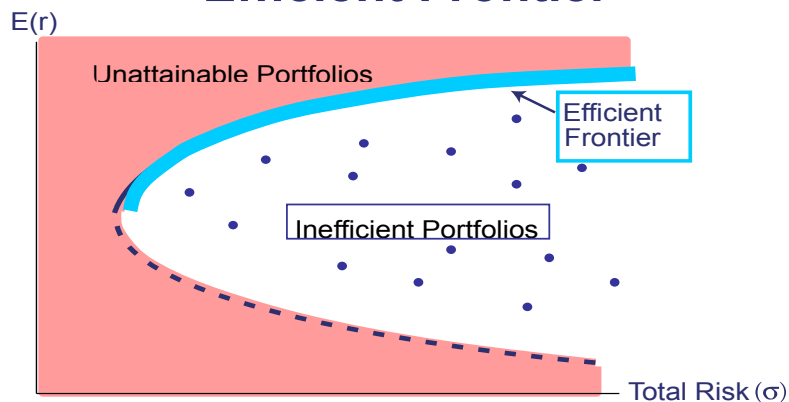
The Standard Capital Asset Pricing Model

Topic 10

Efficient Frontier Background

Efficient frontier is the set of portfolios among all the possible portfolios of combinations of individual risky assets that offers the highest expected return for each level of risk (standard deviation)

Efficient Frontier



The efficient frontier is the upper boundary of the set of all possible portfolio risk/return combinations.

Example: Efficient Portfolios

Which of the following portfolios is *not* on the Markowitz efficient frontier?

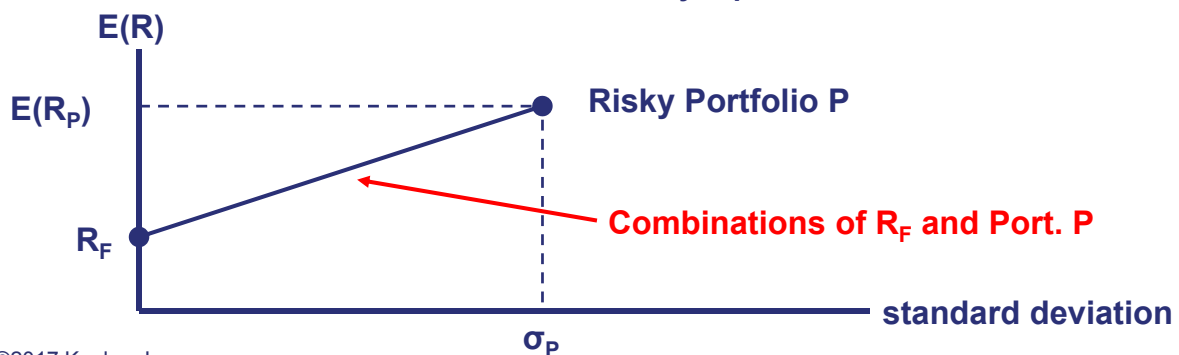
<u>Portfolio</u>	<u>Expected Return</u>	<u>Std. Dev.</u>
A	10%	12%
B	12%	16%
C	14%	15%

B has lower return and higher standard deviation compared to C. B can't be efficient.

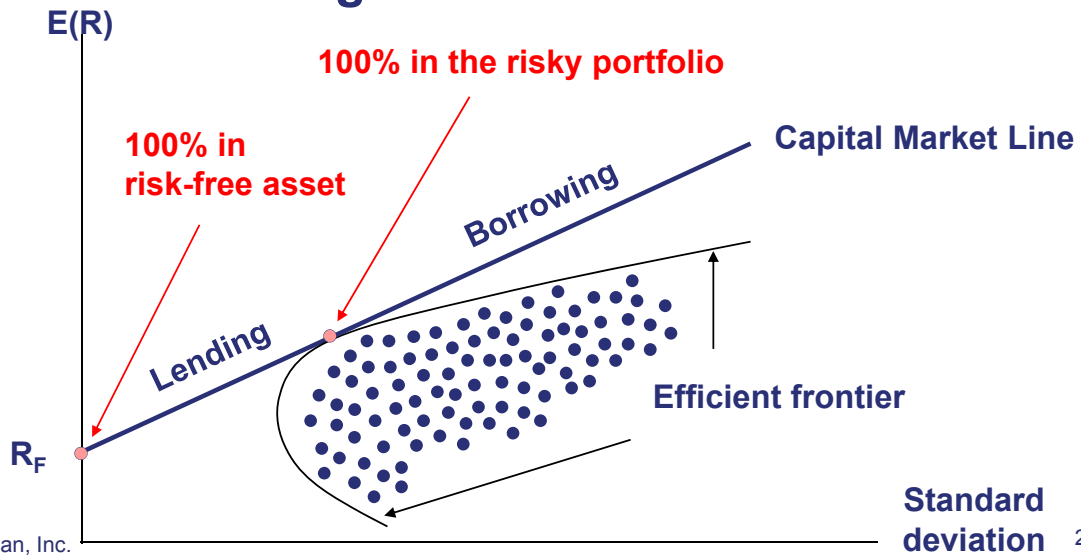
Combining Risk-Free and Risky Assets

$$\text{Portfolio expected return} = W_{\text{risky}} R_p + W_{\text{RF}} R_F$$

$$\text{Portfolio standard deviation} = W_{\text{risky}} \sigma_p$$



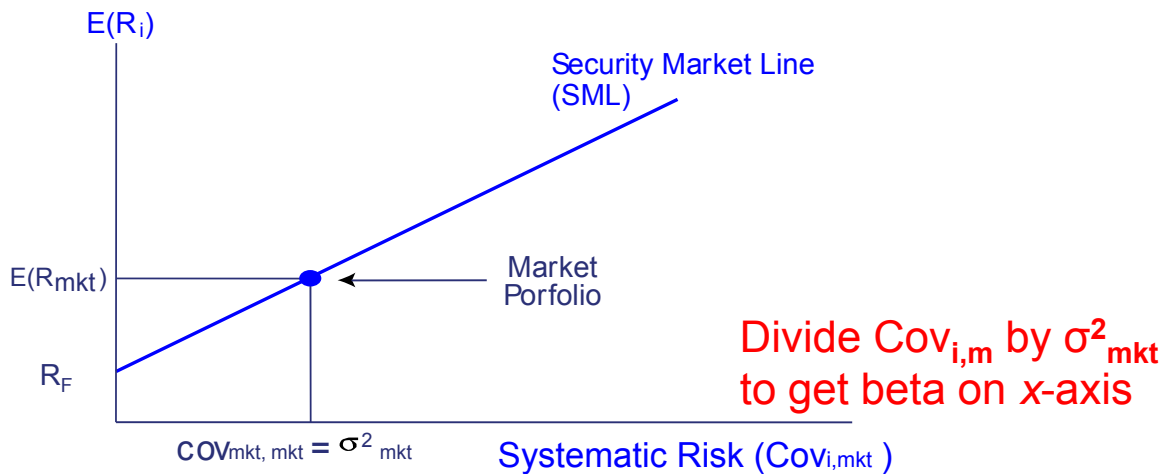
Adding a Risk-Free Asset



Assumptions of Capital Market Theory

- Investors use **mean-variance** framework
- Unlimited **lending and borrowing** at R_F
- Homogeneous **expectations**
- **One-period** time horizon
- **Divisible** assets
- **Frictionless** markets
- No inflation and **unchanging interest rates**
- Capital markets are in **equilibrium**

Security Market Line (SML)



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29

The Capital Asset Pricing Model

SML equation:

$$E(R_i) = R_F + \beta_i [E(R_{mkt}) - R_F]$$

Beta is a standardized measure of systematic risk, beta of the market portfolio is 1

Beta measures the covariance of an asset's returns with returns on the market portfolio

Calculating beta of asset 'i': $\beta_i = \frac{Cov_{i,mkt}}{\sigma^2_{mkt}}$

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30

Capital Asset Pricing Model (CAPM)

- CAPM: The expected return on an asset based (only) on the asset's systematic risk or beta
- CAPM is also used to determine the required return on an asset based on the asset's systematic risk (beta)
- Required return and expected return are the same in equilibrium

Forecast Returns and the CAPM – Example

Stock	Price Today	E (price) in 1 year	E (dividend) in 1 year	Beta
A	\$25	\$27	\$1.00	1.0
B	40	45	2.00	0.8
C	15	17	0.50	1.2

Are these stocks overpriced, underpriced, or at their equilibrium prices?

Show where they plot on the SML graph.

Forecast Returns and the CAPM – Example

Stock	Price Today	E (price) in 1 year	E (dividend) in 1 year	Beta
A	\$25	\$27	\$1.00	1.0
B	40	45	2.00	0.8
C	15	17	0.50	1.2

$$R_F = 7\%$$

$$E[R_{mkt}] = 15\%$$

MRP

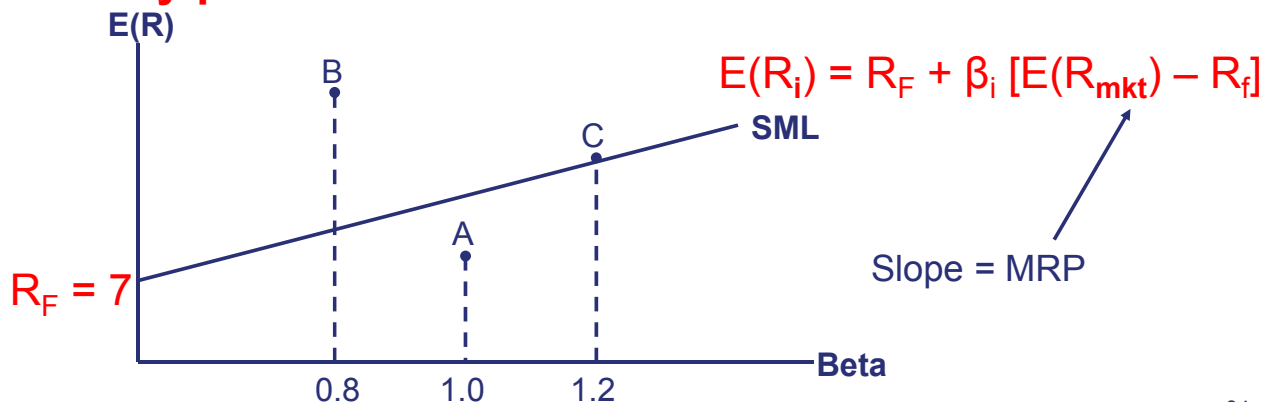
Stock	Forecast Return	Required Return
A	$(27-25+1)/25 = 12.0\%$	$0.07+1.0(0.15-0.07) = 15.0\%$
B	$(45-40+2)/40 = 17.5\%$	$0.07+0.8(0.15-0.07) = 13.4\%$
C	$(17-15+0.50)/15 = 16.6\%$	$0.07+1.2(0.15-0.07) = 16.6\%$

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33

Forecast Returns and the CAPM – Example

When securities are priced at equilibrium values, they plot on the SML.



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34

Forecast Returns and the CAPM

Stock	Forecast Return	Required Return
A	12.0%	15.0%
B	17.5%	13.4%
C	16.6%	16.6%

Stock A is overvalued (sell it or sell it short)

Stock B is undervalued (buy it)

Stock C is properly valued (indifferent)

SML Pricing – Example

$$E[R_{\text{mkt}}] = 15\%; E[R_{\text{Stock X}}] = 17\%; R_F = 8\%$$

$$\text{Stock X's beta} = 1.25$$

Using this data and the capital asset pricing model, which of the following statements about X's stock is true? Stock X is:

- A. overvalued by 1.75 percentage points.
- B. properly valued.
- C. undervalued by 0.25 percentage points.

$$8 + 1.25(15 - 8) = 16.75 \quad 17 - 16.75 = 0.25$$

Applying the CAPM to Performance Measurement

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Calculate Book 1, LO 11.1

Measures of Performance

- Modern portfolio theory and the CAPM are built upon the link between risk and return
- Three measures exist to assess an asset's or portfolio's return with respect to its risk
 - **Treynor measure** = risk premium divided by beta, or **systematic risk**
 - **Sharpe measure** = risk premium divided by standard deviation, or **total risk**
 - **Jensen's alpha** = asset's excess return over the return predicted by the CAPM

Measures of Performance (continued)

$$\text{Treynor} = \frac{E(R_P) - R_F}{\beta_P}$$

$$\text{Sharpe} = \frac{E(R_P) - R_F}{\sigma_P}$$

$$\text{Jensen's alpha, } \alpha_p = E(R_P) - \{R_F + \beta_P [E(R_M) - R_F]\}$$

Measures of Performance (continued)

- In all three cases, the higher the better
- Investors can apply Sharpe measure to all portfolios because it uses total risk
- Treynor measure is more appropriate for comparing well-diversified portfolios
- Jensen's alpha is most appropriate for comparing portfolios that have the same beta

Tracking Error

- **Tracking error** is the standard deviation of the difference between the portfolio return and the benchmark return
- Typically, the manager must keep the tracking error below a stated threshold

$$\text{Tracking error} = \sigma_{R_P - R_B}$$

Information Ratio

- **Information ratio** (IR) is the alpha of the managed portfolio relative to its benchmark divided by the tracking error
- It is essentially a measure of how well the manager has acquired and used information compared to the average manager

$$IR = \frac{E(R_P) - E(R_B)}{\text{tracking error}}$$

Tracking Error and Information Ratio

- Compute the tracking error and the information ratio for the following returns of a portfolio and its benchmark:

Portfolio returns (%)	Benchmark returns (%)
9.5	8.7
8.0	7.8
-2.2	-3.4
11	12.4

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Tracking Error and Information Ratio (continued)

- First, compute the alpha as the difference between the returns:

Portfolio returns (%)	Benchmark returns (%)	Alpha
9.5	8.7	0.8
8.0	7.8	0.2
-2.2	-3.4	1.2
11	12.4	-1.4

- Click 2nd Data (on TI BA II+) and input the alphas as Xs. In 2nd STAT (1-V function), X-bar is average alpha = 0.2 and Sx is tracking error = 1.143.
- The information ratio is alpha/tracking error = $0.2/1.143 = 0.175$

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Sortino Ratio

- The **Sortino ratio** can be interpreted as a variation of the Sharpe ratio that is more appropriate for a case where returns are not symmetric
- We replace the risk-free rate with a minimum acceptable return, denoted R_{\min} , and we replace the standard deviation with a type of semi-variance
- A semi-variance measures the variability of only those returns that fall below the minimum acceptable return

Sortino Ratio (continued)

- Letting R_{\min} denote the minimal acceptable return and semi-standard deviation represent the risk measure:

$$\text{Sortino ratio} = \frac{E(R_P) - R_{\min}}{\text{semi-standard deviation}}$$

Sample Exam Question

A portfolio manager received a report on his fund's performance during 2011. According to the report, the portfolio return was 2.5% with a standard deviation of 21% and a beta of 1.2. The risk-free rate over this period was 3.5%, the semi-standard deviation of the portfolio was 16%, and the tracking error of the fund was 2%. Compute the difference between the value of the fund's Sortino ratio (assuming the risk-free rate is the minimal acceptable return) and its Sharpe ratio.

- a. 0.563.
- b. 0.347.
- c. -0.053.
- d. -0.015.

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47

Answer

Answer: **d**

Sharpe ratio =

$$[E(R_P) - R_F] / \sigma$$

$$(2.5 - 3.5) / 21 = -0.0476$$

Sortino ratio =

$$[E(R_P) - R_{\min}] / (\text{Semi-standard deviation})$$

$$(2.5 - 3.5) / 16 = -0.0625$$

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48

Sample Exam Question

The information ratio of Large Hedge Fund is equal to 2 when using the S&P 500 as the benchmark index. During the same time period, standard deviation of the hedge fund returns is 4%, the hedge fund's Sharpe ratio is 3, and its tracking error against the S&P 500 is 6%. Calculate the return for the S&P 500 during the time period assuming the risk-free rate is 3%.

- a. 3%.
- b. 7%.
- c. 11%.
- d. 15%.

Answer

Answer: a

Sharpe ratio = 3

$$(\text{Portfolio return} - \text{risk-free rate}) / \text{SD of fund} = 3$$

$$(\text{Portfolio return} - 3\%) / 4\% = 3$$

$$\text{Portfolio return} = 15\%$$

Information ratio = 2

$$(\text{Portfolio return} - \text{S\&P return}) / \text{Tracking error} = 2$$

$$(15\% - \text{S\&P return}) / 6\% = 2$$

$$\text{S\&P 500 return} = 3\%$$

Sample Exam Question

Portfolio Y has a beta of 0.8 and an expected return of 10%. The market risk premium is 6.45% and the risk-free rate is 3.7%. Jensen's Alpha measure for this portfolio is closest to:

- a. 10%.
- b. 8%.
- c. 3%.
- d. 1%.

Answer

Answer: **d**

Jensen's alpha

$$= E(R_P) - [R_F + \beta_P(E(R_M) - R_F)]$$

$$= 10\% - [3.7\% + 0.8(6.45\%)]$$

$$= 1.14\%$$

Arbitrage Pricing Theory and Multifactor Models of Risk and Return

Topic 12



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Book 1, LO 12.1

Multifactor Model of Risk and Return

$$R_i = E(R_i) + \beta_{i1}F_1 + \beta_{i2}F_2 + \dots + \beta_{ik}F_k + e_i$$

where:

R_i = return on stock i

$E(R_i)$ = expected return for stock i

β_{ij} = j th factor beta for stock i (i.e., sensitivity of stock return to 1-unit change in the factor)

F_j = deviation of macroeconomic factor j from its expected value (i.e., portion of the stock's return that is unexplained by macro factors)

e_i = firm-specific return for stock i

Hedging Exposures to Multiple Factors

- Consider an investor who manages a portfolio with the following factor betas:
 - GDP beta = 0.50
 - Inflation risk beta = 0.25
- Assume the investor wishes to hedge away GDP factor risk, yet maintain inflation risk exposure
- The investor should combine the original portfolio with a 50% short position in the **GDP factor portfolio** (see next slide)

Hedging Exposures to Multiple Factors (continued)

- **Factor portfolios**: well-diversified portfolios with betas equal to one for a single risk factor and betas equal to zero on all remaining factors
- In this example, the GDP factor beta on the 50% short position in the GDP factor portfolio equals -0.50 , which perfectly offsets the 0.50 GDP factor beta on the original portfolio
- The combined long and short positions hedge away GDP risk but retain inflation risk exposure

APT Assumptions

Arbitrage pricing theory (APT) assumes that:

- *Returns are derived from a multifactor model:* APT provides little practical guidance for the identification of the risk factors in the model (major weakness of the model)
- *Unsystematic risk is completely diversified away*
- *No arbitrage opportunities exist:* implies that investors will undertake infinitely large positions (long and short) to exploit any perceived mispricing, causing asset prices to adjust immediately to their equilibrium values

The Arbitrage Pricing Theory Model

- APT assumes a market model of the form:

$$R_i = R_F + (\beta_1 \times F_1) + (\beta_2 \times F_2) + \dots$$

Diagram illustrating the components of the APT model equation:

- Risk-free rate** points to R_F
- Factor risk premiums** points to $(\beta_1 \times F_1)$ and $(\beta_2 \times F_2)$
- Factor sensitivities** points to β_1 and β_2

- The CAPM model is a special case of APT with only one factor—the market risk premium

Calculate Required Return: APT

- **Problem:** Factors NOT specified by theory
- Examples of common “accepted” factors:
 1. Investor confidence risk
 2. Time horizon risk
 3. Inflation risk
 4. Business-cycle risk
 5. Market-timing risk
- You will be given the factors on exam day.

Do not
memorize

Calculate Required Return: APT Example

- **Assume:**
 - Risk-free rate of 3%
 - **Three** factors and sensitivities

	Risk Premium	Sensitivity
<i>Investor confidence risk</i>	2%	1.1
<i>Time horizon risk</i>	4%	1.2
<i>Inflation risk</i>	3%	0.8

$$R_i = 3\% + 1.1(2\%) + 1.2(4\%) + 0.8(3\%) = \mathbf{12.4\%}$$

Fama-French Three-Factor Model

- APT offers no guidance as to the identification of the appropriate risk factors
- In contrast, the Fama-French model identifies the factors
- In addition to the market return factor, the Fama-French three-factor model specifies the following two factors:
 - SMB (small minus big): firm size factor
 - HML (high minus low): book-to-market factor

Fama-French Three-Factor Model (continued)

Fama-French three-factor model equation:

$$R_i - R_F = \alpha_i + \beta_{i,M}(R_M - R_F) + \beta_{i,SMB}SMB + \beta_{i,HML}HML + e_i$$

- Intercept equals abnormal performance of the asset after controlling for its exposures to the market, firm size, and book-to-market factors
- SMB and HML factors are used because historical returns are higher on small versus big firms and on high versus low book-to-market firms

Principles for Effective Data Aggregation and Risk Reporting

Topic 13



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Book 1, LO 13.1

Benefits of Risk Data Aggregation

- Effective risk data aggregation and reporting benefits include:
 - An increased ability to anticipate problems
 - Enhanced ability to identify alternative routes to restore financial health in times of stress
 - Improved resolvability in the event of bank stress or failure
 - An enhanced ability to make strategic decisions, increasing the bank's efficiency, reducing the chance of loss, and ultimately increasing bank profitability

Governance

- The Basel Committee on Banking Supervision has set forth principles for effective risk data aggregation and risk reporting
- The governance principle (Principle 1) suggests that risk data aggregation should be part of the bank's overall risk management framework
- The board and senior management should assure that adequate resources are devoted to risk data aggregation and reporting

Data Architecture and IT Infrastructure

- The data architecture and IT infrastructure principle (Principle 2) states that a bank should design, build, and maintain data architecture and IT infrastructure which fully supports its risk data aggregation capabilities and risk reporting practices during normal times and times of stress, while still meeting the other principles
- It stresses that banks should devote considerable financial and human resources to risk data aggregation and reporting

Effective Risk Data Aggregation

- Banks should ensure that the data is accurate and has integrity (Principle 3), is complete (Principle 4), is timely (Principle 5), and is adaptable to the end user (Principle 6)
- In addition, the bank should not have high standards for one principle at the expense of another principle
- Aggregated risk data should exhibit all of the features together, not in isolation

Effective Risk Reporting Practices

- Risk reports should be accurate (Principle 7), comprehensive (Principle 8), and clear and useful (Principle 9)
- Principle 10 states that reports should be “appropriately frequent” (frequency depends on the role of the recipient)
- Principle 11 states that reports should be distributed to relevant parties in a timely fashion while maintaining confidentiality

GARP Code of Conduct

Topic 14



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Book 1, LO 14.1

Code of Conduct

- The **GARP Code of Conduct** contains a set of key principles designed to support financial risk management practices
- Developed for the FRM program as well as other certification programs administered by GARP
- All GARP members (including FRM candidates) are expected to abide by the principles outlined in the Code and are subject to consequences for violating any parts of the Code

Code of Conduct (continued)

- When encountering a situation not specifically outlined in the Code, members are always expected to act in an ethical fashion
- Acting with prudence in all situations related to the profession will uphold the integrity of the risk management field as well as risk management practitioners

Code of Conduct (continued)

- The Code of Conduct stresses ethical behavior in two areas: (1) **principles** and (2) **professional standards**
- The principles section addresses: (1) professional integrity and ethical conduct, (2) conflicts of interest, and (3) confidentiality
- The professional standards section addresses: (1) fundamental responsibilities and (2) adherence to generally accepted practices in risk management

Professional Integrity and Ethical Conduct

- GARP members shall:
 - Act professionally, ethically, and with integrity
 - Exercise reasonable judgment while maintaining independence
 - Not knowingly misrepresent
 - Not engage in any professional conduct involving dishonesty or deception
 - Not compromise the integrity of GARP or the FRM designation

Conflict of Interest

- GARP members shall:
 - Act fairly in all situations and must fully disclose any actual or potential conflict
 - Make full and fair disclosure of all matters that could reasonably be expected to impair independence and objectivity

Confidentiality

- GARP members:
 - Shall not make use of confidential information for inappropriate purposes
 - Must not use confidential information for personal benefit

Fundamental Responsibilities

- GARP members shall:
 - Comply with all applicable laws, rules, and regulations (including this Code)
 - Understand the needs and complexity of their employer or client
 - Be diligent about not overstating certainty of results
 - Clearly disclose the relevant limits of their knowledge

Best Practices

- GARP members shall:
 - Execute all services with diligence
 - Be familiar with current generally accepted risk management practices
 - Ensure that communications do not contain false information
 - Make a distinction between fact and opinion

Violations of the Code of Conduct

- All GARP members are expected to act in accordance with the GARP Code of Conduct as well as any local laws and regulations that pertain to the risk management profession
- If the Code and certain laws conflict, then laws and regulations will take priority
- Violations of the Code of Conduct may result in temporary suspension or permanent removal from GARP membership or right to use FRM designation

Foundations of Risk Management Sample Problems and Solutions

2017 FRM Exam Part I



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1. Credit Risk

- Which of the following is not usually associated with credit risk?
 - A. Downgrade Risk
 - B. Interest Rate Risk
 - C. Default Risk
 - D. Settlement Risk

1. Answer

- Answer: **B**
- Interest Rate Risk measures systematic exposure of an instrument and is usually associated with market risk, not credit risk.

2. Capital Asset Pricing Model

- The expected rate of return on a stock is 1.5 times the 16% expected rate of return from the market.
- What is the beta of the stock if the risk-free rate is 8%?

2. Answer

According to the CAPM for any single security or portfolio of securities i , the expected return in equilibrium is:

$$E(R_i) = R_F + \beta_i[E(R_M) - R_F]$$

$$24\% = 8\% + \beta(16\% - 8\%)$$

$$24\% = 8\% + \beta(8\%)$$

$$\text{Beta is then equal to } = \beta = 16 / 8 = 2$$

3. Sharpe and Treynor Measures

- Performance data for an actively managed portfolio and the S&P 500 Index is reported as:

	Portfolio	S&P 500
Return	10%	8%
Standard deviation	6%	5%
Beta	1.1	1.0

- Risk-free rate = 3%
- Determine the Sharpe and Treynor measures for the actively managed portfolio.

3. Answer

Sharpe measure:

$$= (0.10 - 0.03) / 0.06 = 1.17$$

Treynor measure:

$$= (0.10 - 0.03) / 1.1 = 0.064$$

4. Alpha

For a given portfolio, the expected return is 9% with a standard deviation of 16%. The beta of the portfolio is 0.8. The expected return of the market is 12% with a standard deviation of 20%. The risk-free rate is 3%.

What is the portfolio's alpha?

4. Answer

Alpha, α_p

$$= E(R_p) - \{R_F + [E(R_M) - R_F]\beta_p\}$$

$$= 0.09 - [0.03 + (0.12 - 0.03) \times 0.8] = -0.012$$

$$= -1.2\%$$

5. Information Ratio

- Portfolio X return = 11%
- Benchmark return = 10%
- Tracking error volatility = 6%
- Risk-free rate = 4%
- What is Portfolio X's information ratio?

5. Answer

- $IR = \text{alpha} / \text{tracking error}$
- $IR = (11\% - 10\%) / 6\% = 0.167$
- The information ratio is a measure of how well the manager has acquired and used information compared to the average manager.

6. Sortino Ratio

- Portfolio X return = 11%
- Benchmark return = 10%
- Tracking error volatility = 6%
- Minimum acceptable return = 5%
- Risk-free rate = 4%
- Semi-standard deviation = 20%
- What is Portfolio X's Sortino ratio?

6. Answer

The Sortino ratio can be interpreted as a variation of the Sharpe ratio that is more appropriate for a case where returns are not symmetric.

$$\begin{aligned}\text{Sortino ratio} &= \frac{E(R_P) - R_{\min}}{\text{semi-standard deviation}} \\ &= \frac{11\% - 5\%}{20\%} = 0.3\end{aligned}$$

Quantitative Analysis

- Discrete and continuous probability distributions
- Estimating the parameters of distributions
- Population and sample statistics
- Bayesian analysis
- Statistical inference and hypothesis testing
- Estimating correlation and volatility using EWMA and GARCH models
- Correlations and copulas
- Linear regression with single and multiple regressors
- Time series analysis and forecasting
- Simulation methods

Probabilities

Topic 15



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Book 2, LO 15.1

Probability Terminology

- **Random variable:** Uncertain quantity/number
- **Outcome:** Observed value of a random variable
- **Event:** Single outcome or a set of outcomes
- **Mutually exclusive events:** Events that cannot both happen at the same time
- **Exhaustive events:** Include all possible outcomes

Example: Probability Terminology

Consider rolling a 6-sided die.

- The number that comes up is a **random variable**
- Rolling a 4 is an **outcome**
- Rolling a 4 and rolling an even number are **events**
- Rolling a 4 and rolling a 6 are **mutually exclusive events**
- Rolling an even number and rolling an odd number is a set of **mutually exclusive** and **exhaustive events**

Random Variables

- **Continuous random variable:** Number of possible outcomes is infinite, even if lower and upper bounds exist. **Example:** the actual amount of daily rainfall between 0 and 100 inches.
- **Discrete random variable:** Number of possible outcomes can be counted, and for each possible outcome there is a measurable and positive probability. **Example:** the number of days it rains in a given month.

Probability of an Event

- There are two defining properties of probability:
 1. The probability of occurrence of any event is between 0 and 1 (i.e., $0 \leq P(E_i) \leq 1$)
 2. If a set of events is mutually exclusive and exhaustive, the probabilities of those events sum to 1 (i.e., $\sum P(E_i) = 1$)
- The probability of rolling any one of the numbers 1–6 with a fair die is $1/6 = 0.1667 = 16.7\%$

Probability Functions

- A **probability function**, $p(x)$, gives the probability that a discrete random variable will take on the value x [e.g., $p(x) = x / 15$ for $X = \{1,2,3,4,5\} \rightarrow p(3) = 20\%$]
- A **probability density function** (pdf), $f(x)$, can be used to evaluate the probability that a continuous random variable will take on a value within a range
- A **cumulative distribution function** (cdf), $F(x)$, gives the probability that a random variable will be less than or equal to a given value

Discrete Uniform Distribution

- A **discrete uniform distribution** has a finite number of possible outcomes, all of which are equally likely

For example, $p(x) = 0.2$ for $X = \{1,2,3,4,5\}$

$$p(2) = 20\%$$

$$F(3) = 60\%$$

$$\text{Prob}(2 \leq X \leq 4) = 60\%$$

Probability of an Event

- **Unconditional probability** (a.k.a. marginal probability): The probability of an event regardless of past or future occurrence of other events
- **Conditional probability**: The occurrence of one event affects the probability of the occurrence of another event. Using probability notation, “the probability of A *given* the occurrence of B” is expressed as $P(A | B)$, where the vertical bar (|) indicates “given,” or “conditional upon.”

Joint Probability

- The **joint probability** of two events is the probability that they will both occur
- Equals the conditional probability that A will occur given B occurs (a conditional probability) and the probability that B will occur (the unconditional probability of B)

$$P(AB) = P(A | B) \times P(B)$$

Joint Probability (continued)

Example using conditional probability:

$P(\text{interest rates will increase}) = P(I) = 40\%$

$P(\text{recession given a rate increase}) = P(R|I) = 70\%$

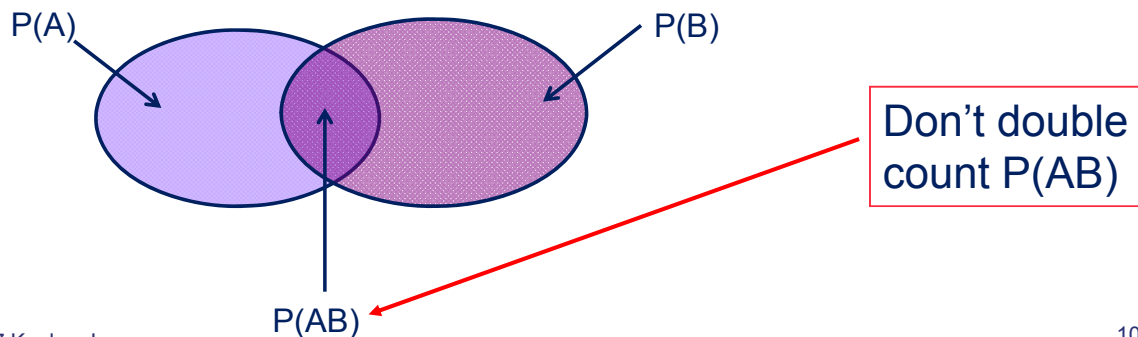
Probability of a recession **and** an increase in rates:

$$P(RI) = P(R|I) \times P(I) = 0.7 \times 0.4 = 28\%$$

Probability That at Least One of Two Events Will Occur

$$P(A \text{ or } B) = P(A) + P(B) - P(AB)$$

We must subtract the joint probability, $P(AB)$



Joint Probability of Any Number of Independent Events

Dependent events: Knowing the outcome of one tells you something about the probability of the other

Independent events: Occurrence of one event does not influence the occurrence of the other. For the joint probability of independent events, just multiply.

Example: Flipping a fair coin, $P(\text{heads}) = 50\%$

The probability of three heads in succession is:

$$0.5 \times 0.5 \times 0.5 = 0.5^3 = 0.125 \text{ or } 12.5\%$$

Probability Matrix

- Joint and unconditional probabilities of independent events can be conveniently summarized using a **probability matrix** (also known as a probability table)

		<i>Interest Rates</i>		
		Increase	No Increase	
<i>Economy</i>	Good	14%	6%	20%
	Normal	20%	30%	50%
	Poor	6%	24%	30%
		40%	60%	100%